Daugavpils Construction Technical School

TYPES OF MARKING AND CALCULATION METHODOLOGY IN GEODESY (*OR IN SURVEYING*)

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Summarized theoretical and practical information on Geodesy.
Can be used as VET training instrument in the field of Road Construction

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CONTENTS

1. INTRODUCTION

The purpose of this document is to summarize and provide information of types of marking and calculation methodology in geodesy to vocational education and training (VET) teachers preparing Geodesy subject lectures and trainings to Road construction and Construction sector learners. The document provides theoretical and practical information in the following basic topics of Geodesy: setting out curves, setting out circular curve, ellipse setting, setting out spiral curve, setting out irregular curve, setting out with tape, setting out with theodolite, setting out with theodolite and tape setting out in plane using other non-surveying instruments. Topics are covered and integrated into different sections of this document. This material can be used during theoretical and practical classes.

The material have been developed in co-operation of Daugavpils Construction Technical School (Latvia) and Marijampole Vocational and Education Training Center (Lithuania), implementing ***project "Types of markings and calculation methodology in geodesy", No 2018-1-LV01-KA116-046913*** funded by *Erasmus+* Programme of the European Union. The information included in this document is result of Daugavpils Construction technical School VET teachers' (Jānis Ancāns, Inga Pujate, Mārtiņš Vilcāns) participation in virtual staff training in Lithuania. Information and examples are designed and summarized during interactive workshops implemented both online and offline.

The information required for the preparation of the document was collected by analysing training programs, methodological recommendations, technical literature in the field, unpublished training instruments designed and used by each partner during classes, and other relevant sources. This material further can be developed and refined up-to-date professional knowledge of geodesy measurements and calculation methodology for vocational education teachers and students.

2. MAIN ELEMENTS OF CURVE.

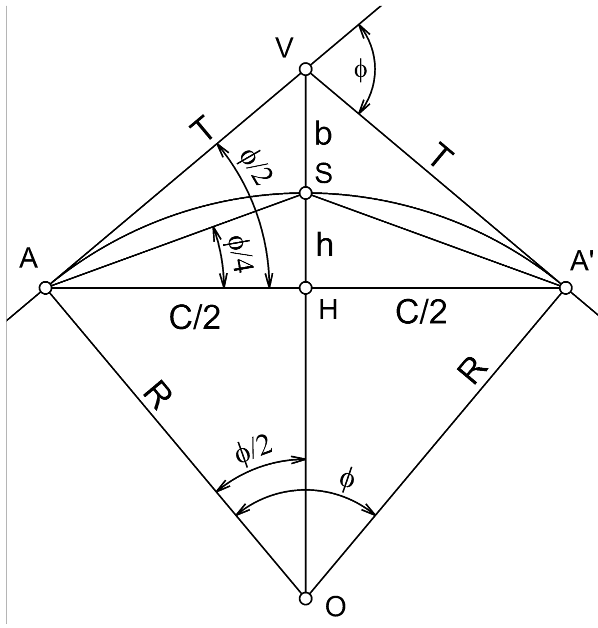


Figure 2.1.

Figure 2.1. represents circular curve joining two tangents. Intersection angle ϕ between two tangents is measured in the field or on the map. The radius R or the curve is selected to fit the topography and proposed operating conditions on the line. The line OV bisects the angles at V and at O, bisects the chord AHA' and the arc ASA' and is perpendicular to the chord AA' at H. From figure ∠AOA' = ϕ.

$$\frac{T}{R} = \tan\frac{\phi}{2}$$

$$T = |\mathrm{AV}| = |\mathrm{AV'}| = R\tan\frac{\phi}{2} = \text{tangent distance}$$

External distance, the length of bisector

$$b = |\mathrm{VS}| = |\mathrm{OV}| - |\mathrm{OS}| = \frac{R}{\cos(\phi/2)} - R = R\left[\frac{1}{\cos(\phi/2)} - 1\right],$$

using rare trigonometric functions

$$b = R\,\mathrm{exsec}\,\frac{\phi}{2} = R\left(\sec\frac{\phi}{2} - 1\right).$$

Length of curve

$$L = \cup\mathrm{ASA'} = \phi^{rad}R = \frac{\varphi^{\circ}\pi}{180^{\circ}}R\,.$$

From triangle AOH, in which AH = C/2, the chord length

$$C = 2R\sin\frac{\phi}{2} = \text{long chord}$$

$$h = R - R\cos\frac{\phi}{2} = R\left(1 - \cos\frac{\phi}{2}\right)$$ = middle ordinate,

using rare trigonometric functions

$$h = R\,\mathrm{versin}\frac{\phi}{2}.$$

From triangle AVH, in which ∠VAH = ϕ/2 and |AH| = C/2,

$$\frac{C}{2} = T\cos\frac{\phi}{2}$$

$$C = 2R\sin\frac{\phi}{2}$$ = long chord.

From triangle ASH, in which ∠SAH = ϕ/4

$$h = \frac{C}{2}\tan\frac{\phi}{4}$$ = middle ordinate.

The curve is shorter than two tangents and route shortens by

$$D = 2T - L$$

Example.

If given angle ϕ = 80º and radius R = 50 m.

Tangent distance $T = R\tan\frac{\phi}{2}$ = 41.955 m

External distance $b = R\left[\frac{1}{\cos(\phi/2)} - 1\right]$ = 15.270 m

Length of curve $L = \frac{\varphi^\circ \pi}{180^\circ}R$ = 69.813 m

Long chord $C = 2R\sin\frac{\phi}{2}$ = 64.279 m

Middle ordinate $h = R\left(1 - \cos\frac{\phi}{2}\right)$ = 11.698 m

Route shortens by $D = 2T - L$ = 14.097 m

3. SETTING OUT CIRCULAR CURVE USING TAPE. ORTHOGONAL OFFSETS TO TANGENT.

Origin of orthogonal coordinate system is at the beginning of arc at point A. General formula of coordinates of any point on curve:

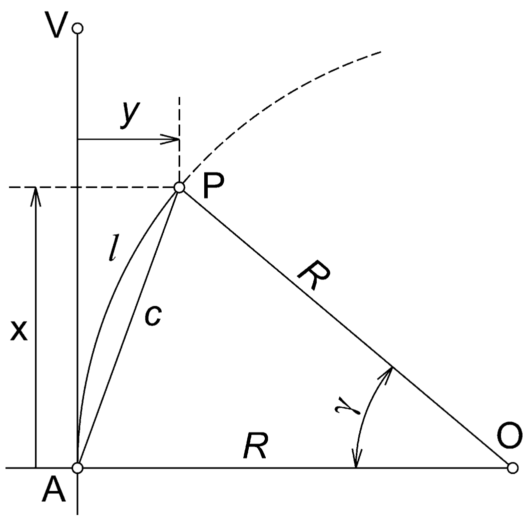


Figure 3.1.

Choose short arc length *l* or short chord length *c* and calculate the central angle γ

$$\gamma^{rad} = \gamma(l) = l / R$$

$$\gamma^{\circ} = \gamma(l) = \frac{l}{R} \cdot \frac{180^{\circ}}{\pi}$$

or from $\sin(\gamma/2) = c/2R$

$$\gamma = \gamma(c) = 2\arcsin\frac{c}{2R}$$

Remember, if taken *l* = *c* then $\gamma(l) < \gamma(c)$.

$$x = R\sin\gamma$$

$$y = R - R\cos\gamma$$

$$y = R(1-\cos\gamma)$$

$$y = 2R\sin^2\frac{\gamma}{2}$$

or

$$y = x\tan\frac{\gamma}{2}$$

Example.

Given angle ϕ = 80º and radius *R* = 50 m.

1. Using arc length *l* = 10 m

   $\gamma^{\circ} = \frac{l}{R} \cdot \frac{180^{\circ}}{\pi} = \frac{10 \cdot 180^{\circ}}{50 \cdot 3.1416}$ = 11.459º

   or using chord length *c* = 10 m

   $\gamma = 2\arcsin\frac{c}{2R} = 2\arcsin\frac{10}{2 \cdot 50}$ =11.478º.
2. In further use more practical is angle calculated by chord *c*, because easy to verify by tape in site.
3. $x = R\sin\gamma = 50\sin(11.478^{\circ})$ = 9.950 m
4. $y = R(1-\cos\gamma)$ = $50 \cdot (1-\sin(11.478^{\circ}))$ = 1.000 m
5. It is first point P_1 with coordinates x_1 = 9.950 and y_2 = 1.000 see next figure.

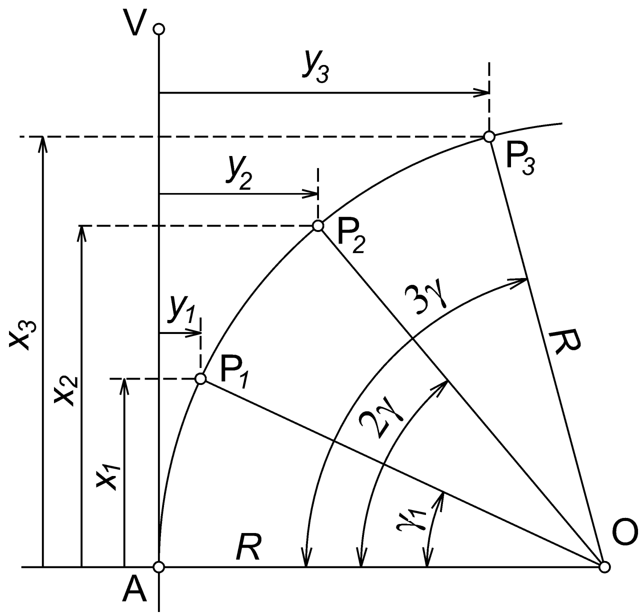


Figure 3.2

6. Multiply angle 2γ for point P_2 and repeat steps 3. and 4. for getting coordinates of next points
7. $x_2 = R\sin(2\gamma) = 50\sin(2\cdot 11.478°)$ = 19.502 m
8. $y = R(1-\cos(2\gamma))$ = $50\cdot(1-\sin(2\cdot 11.478°))$ = 3.960 m
9. Repeat steps 6 to 7 until middle of curve while central angle is not exceeding $\phi/2$.
10 Then lay out all the rest points of curve from endpoint A' reversely, but using the same coordinates.

Table 3.1: Values to be lay out

Point *n*	angle $n\cdot\gamma$ [º]	*x* [m]	*y* [m]
1	11.478	9.950	1.000
2	22.957	19.502	3.960
3	34.435	28.274	8.762
4	45.913	35.914	15.213

4. SETTING OUT CIRCULAR CURVE USING TAPE. STEPWISE MIDDLE DISTANCE.

Follow the steps:
Point S = 1.

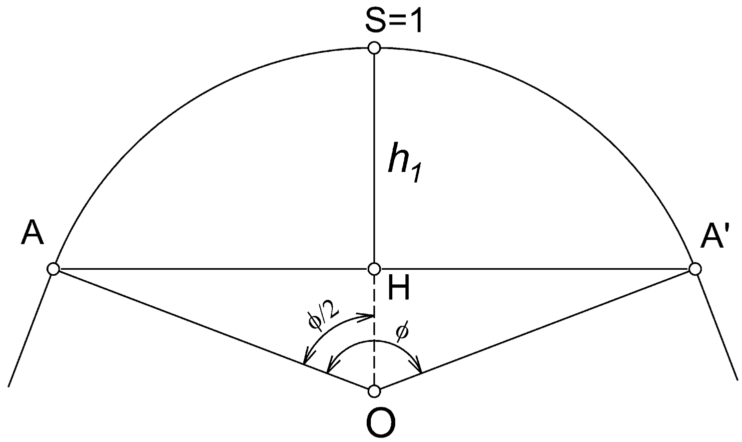


Figure 4.1

1. Calculate middle ordinate $h_1 = R - R\cos\frac{\phi}{2} = R\left(1-\cos\frac{\phi}{2}\right)$
2. Pull the tape between A and A'. That is long chord.
3. From tape's middle point H with other tape lay out the middle ordinate h_1 and fix point 1.
4. Repeat steps 1 to 3 with new half angle $\frac{\phi/2}{2} = \frac{\varphi}{4}$

Point 2.

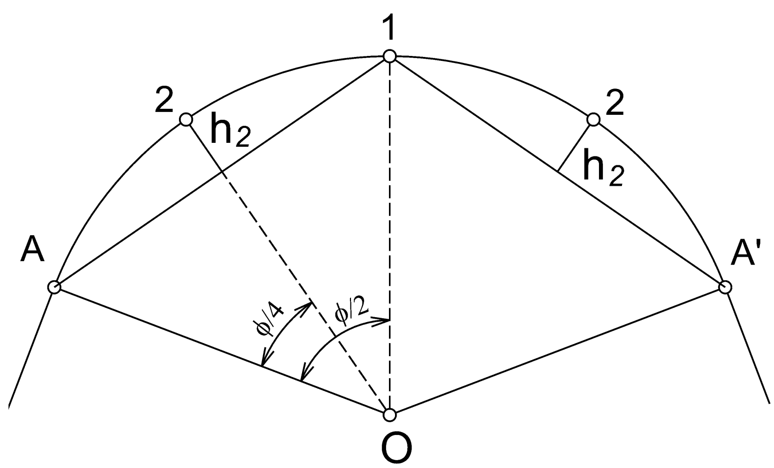


Figure 4.2

1. Calculate middle ordinate $h_2 = R - R\cos\frac{\phi}{4} = R\left(1-\cos\frac{\phi}{4}\right)$.
2. Pull the tape between previously fixed point A and 1.
3. From tape's middle point H with other tape lay out the middle ordinate h_2 and fix point 2.
4. Repeat pulling tape between previously fixed point 1 and A', laying out and fixing point 2.

Point 3.
Repeat dividing angle

$$h_3 = R - R\cos\frac{\phi}{8} = R\left(1 - \cos\frac{\phi}{8}\right)$$

and laying out new midpoints 3.

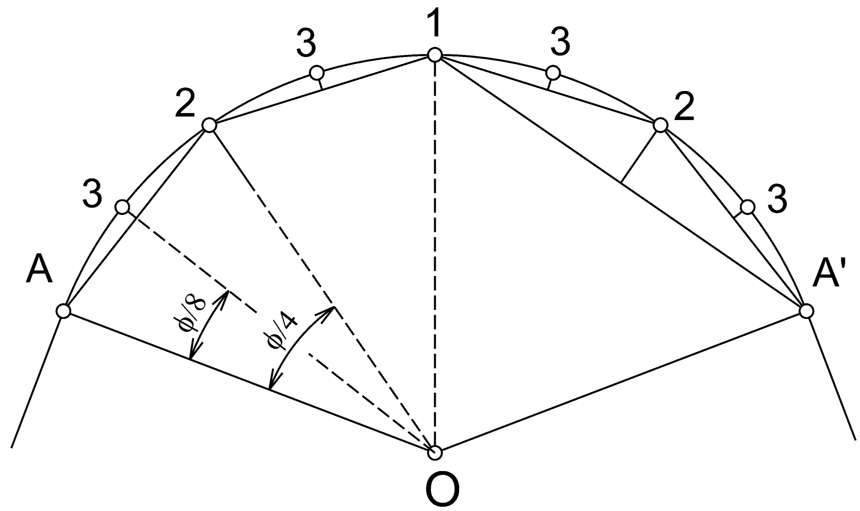


Figure 4.3

The number of points doubles per each step.

Example of stepwise middle distance.
R = 100 m ϕ_0 = 80º

Table 4.1

step i	half angle ϕ_i [º]	$h = R\left(1 - \cos\frac{\phi_{i-1}}{2}\right)$ [m]	points per step
1	40	23.396	1
2	20	6.031	2
3	10	1.519	4
4	5	0.381	8
5	2.5	0.095	16
6	1.25	0.024	32
7	0.625	0.006	64
8	0.313	0.001	128

5. SETTING OUT CIRCULAR CURVE USING THEODOLITE AND TAPE. DEFLECTION ANGLES AND CHORDS.

Setting out Simple Circular Curve by deflection angles and chords from beginning of simple curve.

Curves can be staked out by the use of deflection angles turned at A from the tangent AV to the points P_1, P_2, P_3, ... along the curve together with the use of cords c_1, c_2, c_3, ... measured from station A to points along the curve. Angles can be staked by theodolite and distances by tape.

Formula of long chord $C = 2R\sin\frac{\phi}{2}$ can be adopted to find short chords. Assume the deflection angle γ is given. In chord formula the double angle 2γ is needed at centre or arc. Then the first chord c_1 to first point P_1, will be $c_1 = 2R\sin\frac{2\gamma}{2}$. After simplification $c = 2R\sin\gamma$

and generalization

$$c_n = 2R\sin(n\cdot\gamma)$$

where n – point number and angle multiplier.

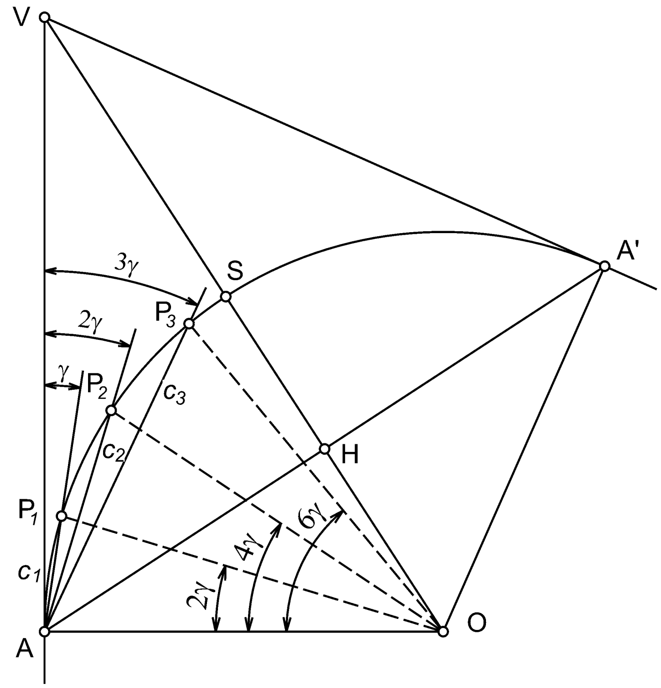


Figure 5.1.

In practice, the specified arc length l is often used. Then like in Tangent offset method the angle

$$\gamma^\circ = \gamma(l) = \frac{l}{R}\cdot\frac{180^\circ}{\pi}.$$

Example.

Given radius R = 50.00 m and arc length l = 20 m.

$$\gamma^\circ = \frac{l}{R}\cdot\frac{180^\circ}{\pi} = \frac{20\cdot180^\circ}{50\cdot3.1416} = 11.4592º.$$

Point P_1

n = 1

$c_n = 2R\sin(n\cdot\gamma)$

$c_1 = 2R\sin(1\cdot\gamma) = 2\cdot50\cdot\sin(11.4592)$ = 19.867 m

Point P_2

n = 2

deflection angle: 2γ = 2· 11.4592 = 22.9183º

chord: $c_2 = 2R\sin(2\cdot\gamma) = 2\cdot50\cdot\sin(22.9183)$ = 38.942 m

Point P_3

$n = 3$

deflection angle: $3\gamma = 3 \cdot 11.4592 = 34.37744º$

chord: $c_3 = 2R\sin(3\cdot\gamma) = 2\cdot 50\cdot\sin(34.3775)$ = 56.464 m

Point P_4

continue multiply deflection angle while $n\gamma < \phi/2$

All above necessary stake out results can be calculated and written

Table 5.1

Point n	deflection angle $\gamma \cdot n$	chord $c = 2R\sin(\gamma \cdot n)$
1	11.4592	19.867
2	22.9183	38.942
3	34.3774	56.464
4	45.8366	71.736
5	continued until $\phi / 2$	continued

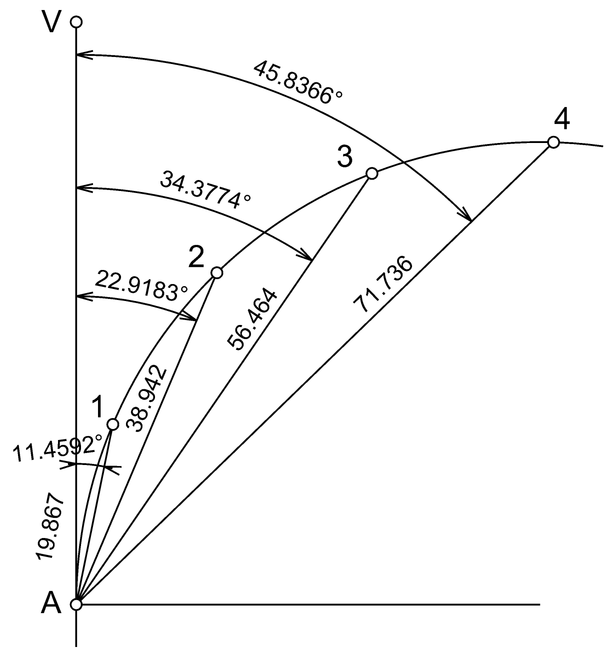


Figure 5.2

6. SETTING OUT CIRCULAR CURVE USING ONLY TWO THEODOLITES. DEFLECTION ANGLES BY THEODOLITES.

As it is described in next figure there are not necessary calculations. Only calculation may be needed to find the first deflection angle γ according to specified arc length or chord length between points like some methods before. Deflection angles can be staked out simultaneously from beginning A and end A' of arc using two theodolites simultaneously.

The synchronisation is required to be sure the operators are staking out the same angle. The third person iteratively searches correct position of points according to instructions of theodolite operators.

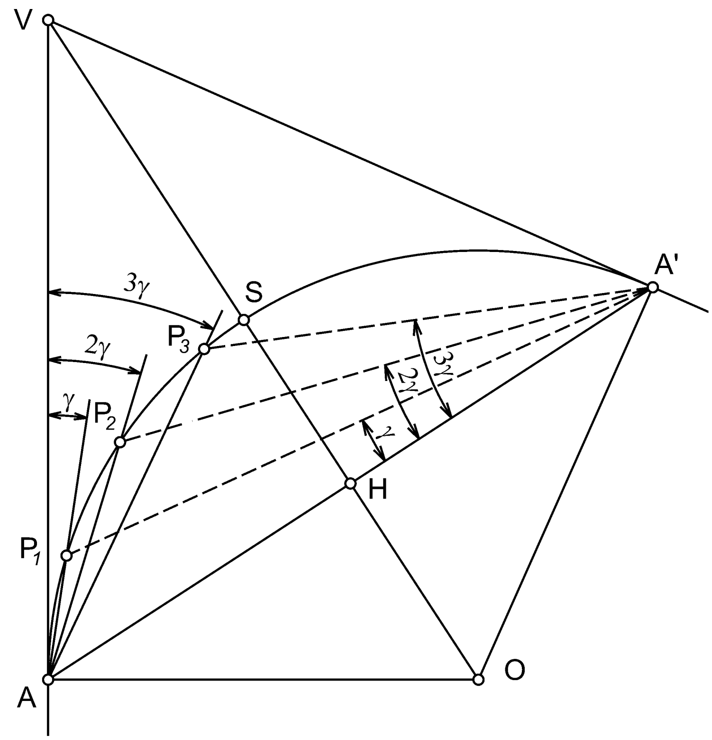


Figure 6.1.

7. SETTING OUT SPIRAL CURVE USING TAPE.

To avoid passenger jam as the vehicles moves from straight to circular curve there is necessary transition curve between straight and circular. This means that radius at the beginning of transition equals to infinity but at the end of transition the radius must fit to radius of circular curve. There are several spiral curves that fulfil these conditions. Most often, clothoid is used, but in stake out fieldwork – the cubic parabola is used.

Clothoid (also known as Cornu spiral or a spiral curve) is described by so-called natural equation, which has following form:

$$a^2 = rl = RL = const$$

where:

a so-called parameter of the spiral curve,
r radius of curvature at any clothoid point,
l arc length measured from the initial point (natural parameter)
R final radius of clothoid
L total length of clothoid

Apart from the curvature radius *r* and the curve length *l*, an important parameter that describes the geometry of the spiral curve is a deflection angle, which will be denoted as ψ:

$$\psi = \frac{l^2}{2a^2} = \frac{l}{2r} = \frac{a^2}{2r^2}$$

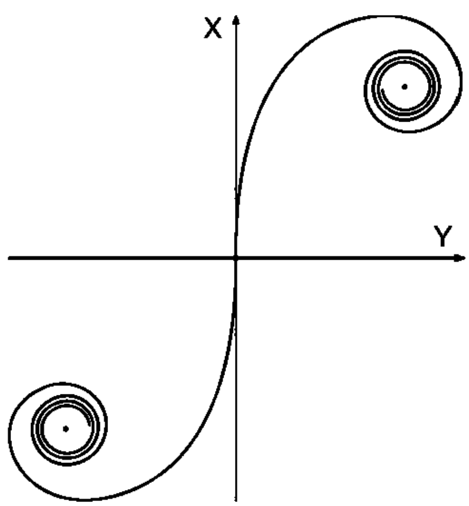


Figure 7.1

Cartesian coordinates of the spiral curve can be expressed using known formulas:

$$x = l\left(1 - \frac{l^4}{40a^4} + \frac{l^8}{3456a^8} - \frac{l^{12}}{599040a^{12}} + \cdots\right)$$

$$y = l\left(\frac{l^2}{6a^2} - \frac{l^6}{336a^6} + \frac{l^{10}}{43340a^{10}} - \frac{l^{14}}{9676800a^{14}} + \cdots\right)$$

or

$$x = l\left(1 - \frac{\psi^2}{10} + \frac{\psi^4}{216} - \frac{\psi^6}{9360} + \cdots\right)$$

$$y = l\left(\frac{\psi}{3} - \frac{\psi^3}{42} + \frac{\psi^5}{1320} - \frac{\psi^7}{75600} + \cdots\right).$$

In stake out fieldwork the cubic parabola is used

$$y = \frac{x^3}{6a^2}.$$

All these formulas are useful for short spirals if ψ do no exceed 45º. For long spirals more complicated formulas are required.

Example.

Given radius R = 50.00 m, total length L = 30 m.

Spiral parameter

$$a^2 = RL = 50 \cdot 30 = 1500$$

Example point 1. The final point of spiral.

$l = L$ = 30 m total spiral length.

by formula

$$x = l\left(1 - \frac{l^4}{40a^4} + \frac{l^8}{3456a^8} - \frac{l^{12}}{599040a^{12}} + \cdots\right)$$

$$x_{30} = 30\left(1 - \frac{30^4}{40 \cdot 1500^2} + \frac{30^8}{3456 \cdot 1500^4} - \frac{30^{12}}{599040 \cdot 1500^6} + \cdots\right) = 29.731 \text{ m}$$

and by formula

$$y = l\left(\frac{l^2}{6a^2} - \frac{l^6}{336a^6} + \frac{l^{10}}{43340a^{10}} - \frac{l^{14}}{9676800a^{14}} + \cdots\right)$$

$$y_{30} = 30\left(\frac{30^2}{6 \cdot 1500} - \frac{30^6}{336 \cdot 1500^3} + \frac{30^{10}}{43340 \cdot 1500^5} - \frac{30^{14}}{9676800 \cdot 1500^7} + \cdots\right) = 3.000 \text{ m}$$

Example point 2. Lets calculate one point outside the spiral just for better visualization only.
Required length of spiral is L = 30 m, but lets take outside l = 40 m.
Using the same formulas and constants

$$x_{40} = 40\left(1 - \frac{40^4}{40 \cdot 1500^2} + \frac{40^8}{3456 \cdot 1500^4} - \frac{40^{12}}{599040 \cdot 1500^6} + \cdots\right) = 38.877 \text{ m}$$

$$y_{40} = 40\left(\frac{40^2}{6 \cdot 1500} - \frac{40^6}{336 \cdot 1500^3} + \frac{40^{10}}{43340 \cdot 1500^5} - \frac{40^{14}}{9676800 \cdot 1500^7} + \cdots\right) = 7.111 \text{ m}$$

Deflection angle

$$\psi_{40} = \frac{l^2}{2a^2} = \frac{40^2}{2 \cdot 1500} = 0.533333^{\text{rad}} = 30.558º$$

Curvature radius

$$r_{40} = a^2 / l = 1500/40 = 37.5 \text{ m}$$

Figure 7.2.

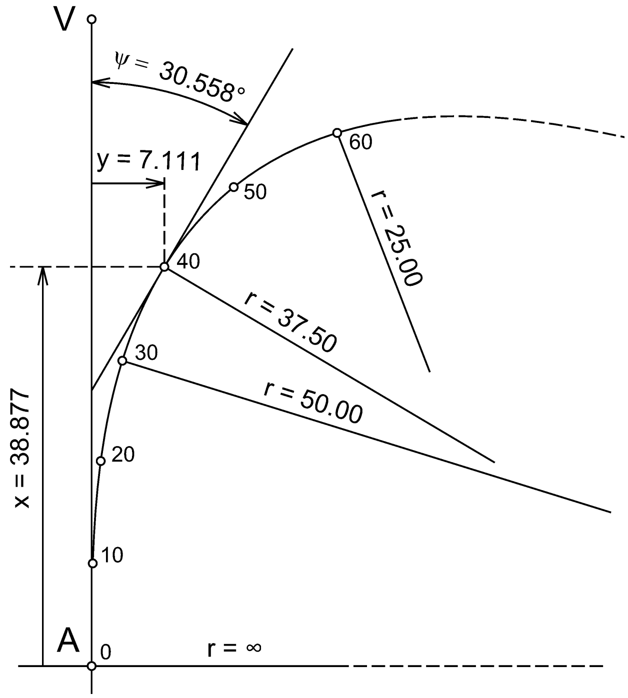


Table 7.1.

Values of spiral in the figure

l	ψ^{rad}	ψ^{O}	*x*	*y*	*r*
10	0.0333	1.910	9.999	0.111	150.0
20	0.1333	7.639	19.964	0.889	75.0
30	0.3000	17.189	29.731	3.000	50.0
40	0.5333	30.558	38.877	7.111	37.5
50	0.8333	47.746	46.638	13.889	30.0
60	1.2000	68.755	51.917	24.000	25.0

8. SETTING OUT IRREGULAR CURVES.

There are known number of methods to lay out irregular lines on area. And there are may be a lot of lost an forgotten methods to do that. The biggest mystery is so called Nazca Lines – a group of very large geoglyphs made in the soil of the Nazca Desert in southern Peru. They were created between 500 BC and AD 500. Thy represent some regular figures, but mainly irregular ones.

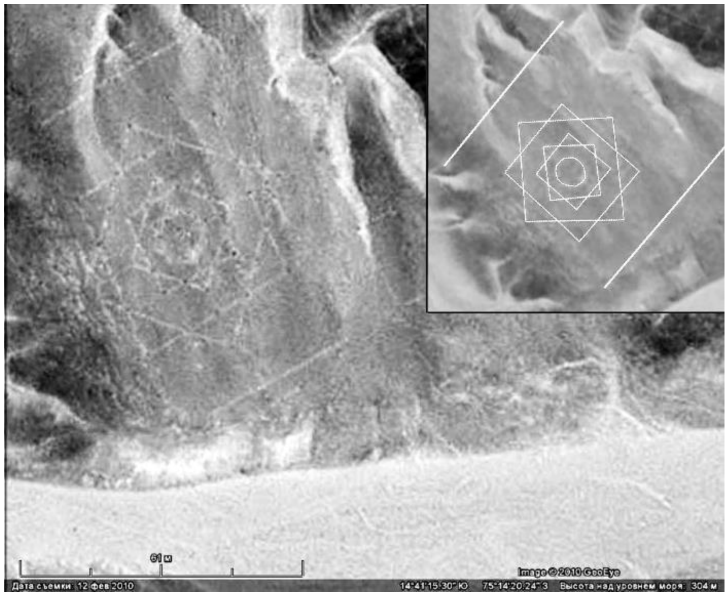


Figure 8.1.

Figure 8.2.

Source: http://www.arcanafactor.org/47-eng/nazca/81-nazca-geoglyphs

But now not about Nazca.

That curve is specially made as a tool for architects and engineers and specially made to have irregular curved edges.

Figure 8.3

Now imagine that to be projected element of landscape design and it is necessary to lay out in landscape. There may be known only general dimensions or just map scale where that is drawn out. So the magnification to natural size is needed. Theoretically there may be fixed photo projector high above and image may be projected on to ground at necessary position and orientation and kept still until all lines are transferred and fixed on ground. But it is philosophically or theoretically.

As it is known methods to lay out regular objects, it is recommended to make irregular objects to be regular or semiregular. The easiest way to regularize is to apply Cartesian coordinate system to lay out object.

Divide and conquer

There are few principal steps to do:

1. Identify dimensions by maximum and minimum. Sometimes may need to identify most comfortable *max* or *min*. Identify origin of coordinate grid system.

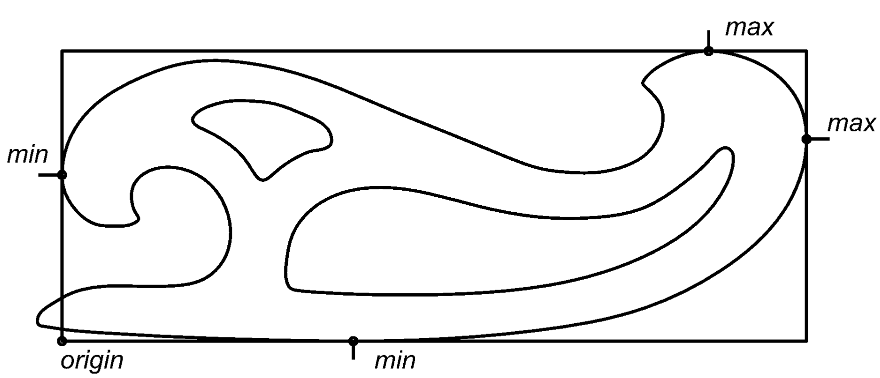


Figure 8.4

2. Draw coordinate grid on project of object. If true size of object is specified in project, use grid interval according to it. If size is not specified exactly, then divide all by halving or quartering. Halving and quartering is recommended, because dividing easily can be continued. Identify intersections of object and grid.

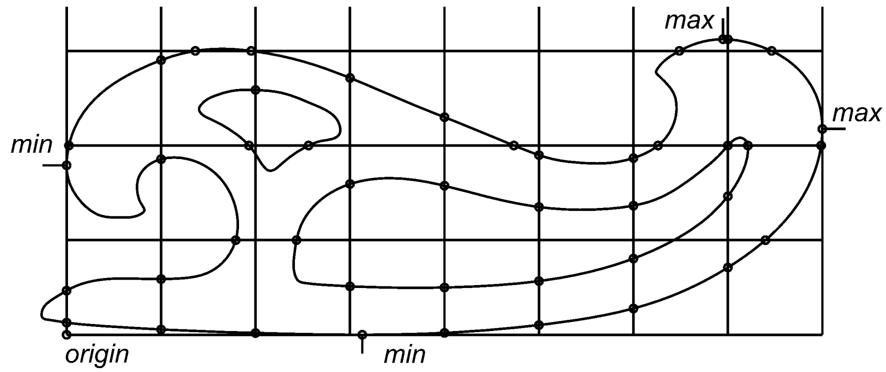
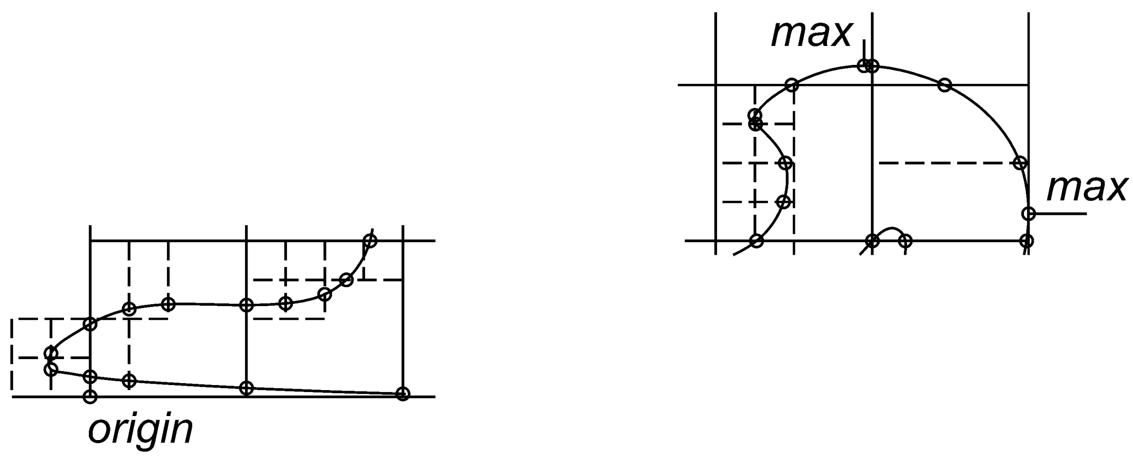


Figure 8.5

3. Repeat these steps on area – fix coordinate grid on site according necessary grid interval and fix identified intersection points on grid edges.

4. Densify grid by dividing previous grid, Measure distances from previous grid to new intersections of densified grid edges and object. If some part of object was outside the existing grid, add some extra grid cells.

Figure 8.6.



5. Densify grid on site by staking out densified grid points. Using densified grid sake out new measured intersections.

6, Connect all intersection points by lines, circular curves or spirals using methods described before.

7. Continue dividing if necessary.

9. SETTING OUT OF ELIPSE USING TAPE.

Lay out of ellipse by staking point according to orthogonal coordinates is complicate because ellipse has no constant radius. Calculation of curve with variable radius require integration and higher mathematics. In practice it is easy to lay out so-called *garden ellipse*.

Following steps required.

1. The rectangle with dimensions 2a and 2b is fixed so the ellipse with semi axes a and b will fit.

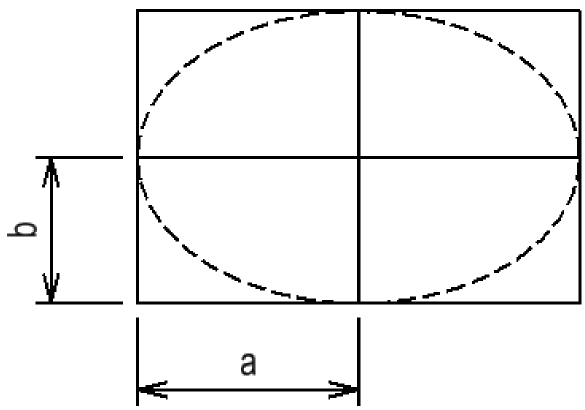


Figure 9.1.

2. The foci must be calculated and located.

As $a^2 = b^2 + c^2$

then $c = \sqrt{a^2 - b^2}$

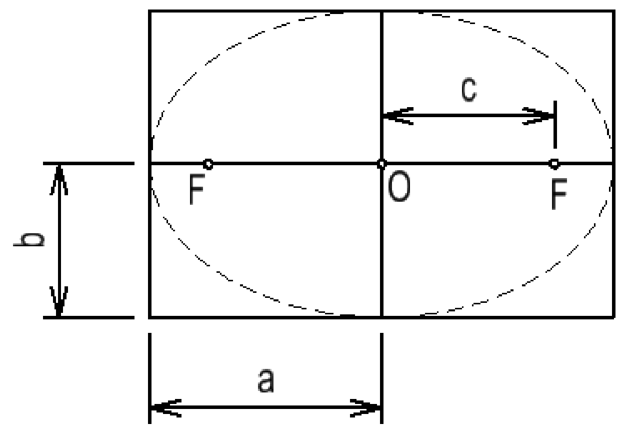


Figure 9.2

3. At the pole of ellipse draw circle with radius $R = a$. Intersections of circle and semi major axis give foci points F.

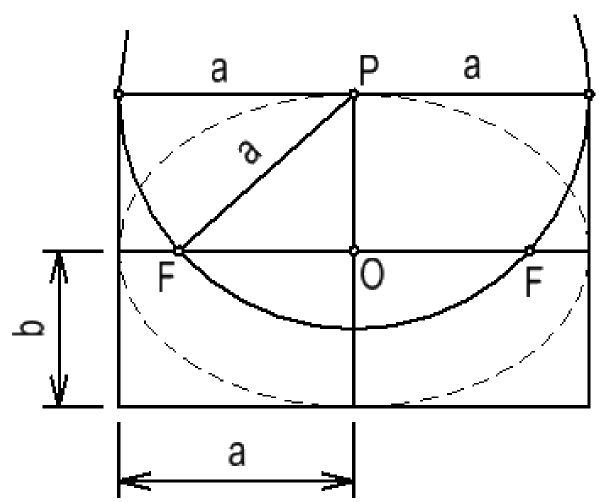
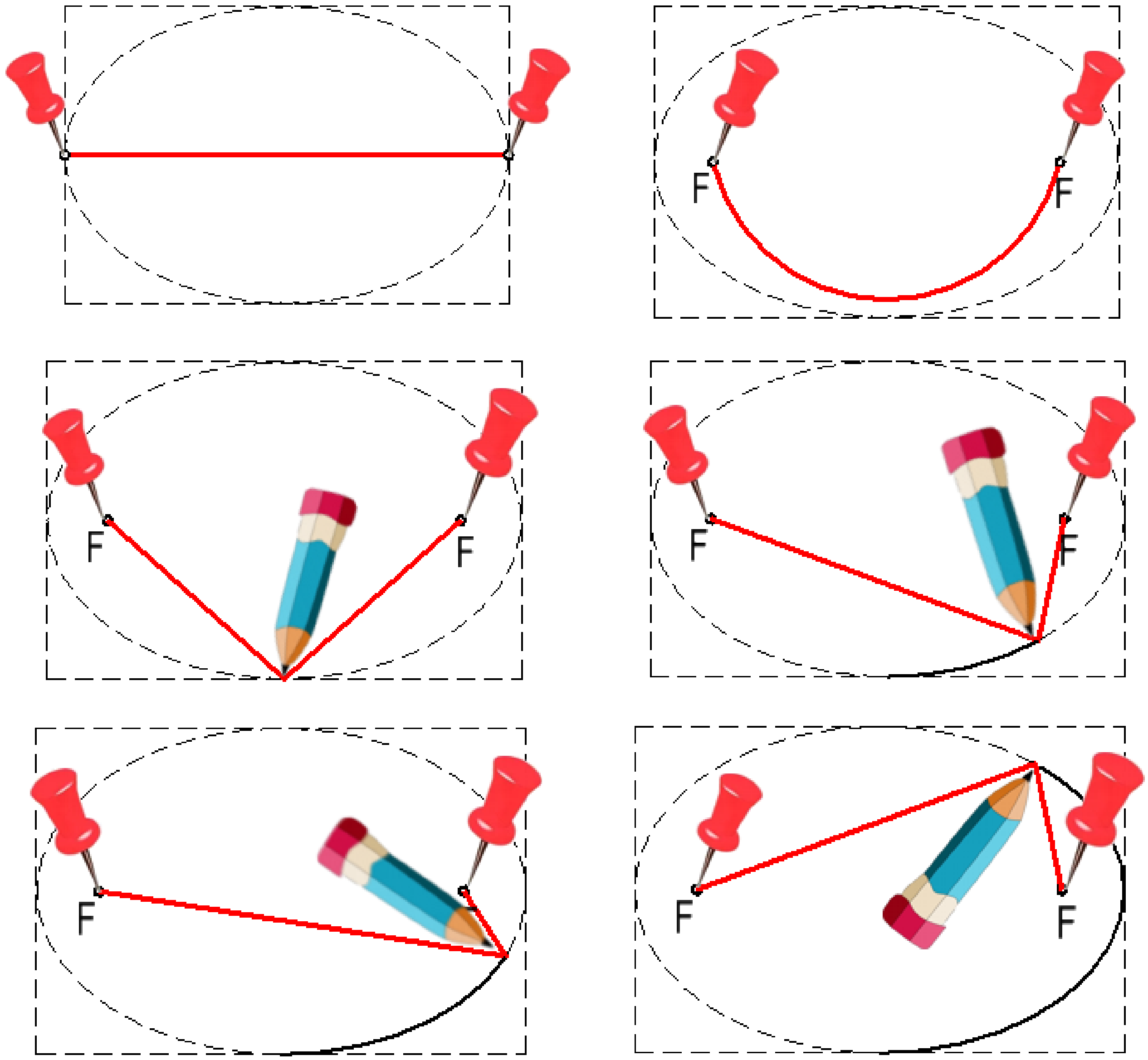


Figure 9.3

4. Finally there are some practical steps in images tells themselves. Good practical example is demonstrated in the historical drama “Agora” year 2009.

Figure 9.4



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8. Curriculum of training program "Road construction and maintenance" with qualification "Road Construction and maintenance worker" Implemented in Marijampole Vocational and Education Training Center (Lithuania), the module "Geodesy Measurement and Marking works" .
9. Unpublished training materials developed by vocational teachers and used within theoretical and practical classes of Geodesy subject (Daugavpils Construction Technical School, Latvia)
10. Unpublished training materials developed by vocational teachers and used within theoretical and practical classes of the Geodesy subject (Marijampole Vocational and Education Training Center, Lithuania).
11. VET teachers summarized information during interactive workshops implemented within Erasmus+ programme project No. 2018-1-LV01-KA116-046913
12. Video materials of Geodetic works. Designed by Marijampole Vocational and Education Training Center, Lithuania.
13. Historical Drama Film "Agora", Directed by Alejandro Amenábar, 2009. Episode about Ellipse. Available: https://www.youtube.com/watch?v=GQDxFNzeDEc [Accessed 14 May 2021].
